Natural Convection Heat Transfer along a Partially Heated Vertical Channel

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الملخص:

هذه الورقة تقدم دراسة معملية للحمل الحراري تبحث في سلوك انتقال الحرارة وانسياب الهواء خلال قناة عمودية مسخنة جزئيا تحت ظروف عملية مختلفة. إرتفاع القناة عبارة عن 1m وعمقها 0.07m ومسخنة من جانب واحد من منتصف القناة بواسطة مسخن حراري مقاسه 1m x 1m. أربع مقادير من الحرارة المضافة (5W, 10W, 15W و20W) أستخدمت ودرجات والحرارة والسرعات قيست عند مواضع مختلفة. قراءات درجات الحرارة أخذت وسجلت لضمان الحصول على حالة الإستقرار الحراري. عند الوصول الى الاستقرار الحراري، المعلومات جمعت وحللت والعلاقات استنتجت. علاقات لابعدية متعلقة بالموضوع أشتقتحيث تم استنتاج أن معامل إنتقال الحرارة ومعدل التدفق الكتلي تعتمد على كمية الحرارة المضافة وبعلاقة اسية للاس 0.33 و 0.3 على التوالي.

الكلمات المفتاحية: الحمل الحراري الطبيعي، التسخين الجزئي، القناة العمودية

ABSTRACT:

This paper presents a natural convection experimental study investigating the heat transfer and airflow behaviour along a partially-heated vertical channel under various operating conditions. The channel had a height of 1m and a depth of 0.07m, and it was symmetrically heated from one side at the middle of the channel using a 0.1m x 0.1m electrical-powered hot plate. Four different heat input values (5W, 10W, 15W, and 20W) were tested, and temperatures and velocities were measured at different locations. Temperature readings were monitored and recorded to ensure the acquisition of a steady-state condition. Once the steady-state heating condition was achieved, data were collected, analyzed, and relationships were established. Dimensionless correlations related to the problem were also derived. It was observed that heat transfer coefficient and mass flow rate were both dependent on the heat input and exhibited a power relationship of 0.33 and 0.3, respectively.

Key words: natural convection, partially heated, vertical channel.

NOMENCLATURE

a, b	Regression coefficiecnts (see equation 9 &11)
A_C	Cross secctiom area (m^2)
A_P	Hot Plate Area ($m^{2)}$
Cp	Specific heat (J/kg.K)
g	Acceleration due to gravity (= 9.81 m/s^2)
H	Channel height (m)
k	Thermal conductivity of air (W/m.K)
$\dot{m}$	Mass airflow rate (kg/s)
Pr	Prandtl number (-)
q_c	Convective Heat to airstream (W/m^2)
q_i	Heat Input (W/m^2)
Ra*	Modified Rayleigh number (-)
Re	Reynolds number (-)
s	Channel depth (m)
T_a	Ambient temperature (°C)
T_i	Inlet temperature (°C)
T_m	Mean air temperature (°C)

T_o	Outlet Temperature (°C)
T_P	Plate temperature (°C)
$\bar{u}$	Mean airflow velocity (m/s)

Greek symbols

β	Expansion coefficient (1/K)
r	Density (kg/m^3)
ν	Kinematic viscosity (m^2/s)

INTRODUCTION:

Natural or free convection is a type of heat transfer that occurs due to natural buoyancy forces in a fluid, such as air or water due to density differences caused by temperature gradients along the fluid flow. One example among many is the natural convection along vertical channels. These types of channels can be found in large scale such as those used in thermal solar heating system, e.g. Trombe wall, or small channels such as those found in electronic cooling components and nuclear reactor. This movement of fluid sets up a natural circulation pattern known as a convection current. The rate of heat transfer due to free convection in these type of channels depends on various factors such as the dimensions of the channel, the temperature difference between the fluid and the adjacent walls, and of course the physical properties of the fluid. The rate of heat transfer due to free convection can be described by the Grashof number (Gr), which is a dimensionless parameter that relates the buoyancy forces to the viscous forces of the fluid. In a small vertical channel, free convection can significantly enhance the rate of heat transfer compared to conduction alone. This is because the convection currents set up by the buoyancy forces increase the mixing of the fluid and reduce the thickness of the thermal boundary layer near the channel walls, leading to higher rates of heat transfer. However, as the channel size decreases, the effect of viscous forces becomes more significant, and the rate of heat transfer due to free convection may start to decrease. Overall, free convection can play an important role in heat transfer in small vertical channels. Understanding the underlying principles of free convection and its impact on heat transfer is essential for designing and optimizing heat transfer systems in various applications.

In regard, many researchers have focused on studying the airflow and the heat transfer characteristics in such channels. For example, the pioneering study of Elenbaas (1942) is considered an important early study of natural convection from isothermal parallel plates, including both experimental data and theoretical treatments. He tested several sizes of heated plates at uniform temperatures and at various distances apart. He found that the heat dissipation, for wide channel depths between the plates, was independent of channel depth, increasing with wall temperature and decreasing slowly with distance up the channel. Several subsequent studies were based directly on his work. Bodoia and Osterle (1962) studied theoretically the development of free convection in viscous fluids between two isothermally heated vertical plates. They derived heat transfer and airflow characteristics in the channel, the development height and dimensionless expression. Levy *et al* (1975) presented experimental data to support the derivations achieved by Elenbaas (1942) and Bodoai and Osterele (1962) regarding the optimum plate spacing for laminar natural convection between two parallel vertical isothermal flat plates. A boundary-layer approximation method was used by Aung (1972), to study fully developed laminar free convection between vertical asymmetrically heated plates. UHF (Uniform Heat Flux) and

UWT (Uniform Wall Temperature) heating mode were considered and related theoretical solutions related were derived. Aung et al (1972) verified the previous theoretical work Aung (1972), by experiment. The channel walls were asymmetrically heated and the thermal boundary condition of UHF and UWT were both studied. In the case of UWT, the theoretical results were verified by the experimental results. Wirtz and Stutzman (1982) collected data of natural convection and presented interferograms of airflow between vertical parallel plates with various channel spacings and heat inputs showing the development of the temperature profiles. Their correlations based on experimental data were in close agreement with the finite difference calculations of Aung et al (1972). Sparrow and Bahrami (1980) reported experimental work results on natural convection between two parallel square plates. The effect of physical parameters and the geometry of the channel itself (aspect ratio s/H) were graphically presented and compared with the work of Elenbaas (1942) with satisfactory agreement in the range of (s/H)Ra >20. Natural convection heat transfer in water ($Pr \cong 5$) was studied experimentally by Azevedo and Sparrow (1985). Various channel aspect ratios (s/H), 0.0437, 0.0656, 0.0856 and 0.109, and inclinations, 0°, 30° and 45°, relative to the vertical, were examined. An empirical equation for Nu was derived based on Ra and (s/H). As an extension of the work of Wirtz and Stutzman (1982) and Aung et al (1972b), Webb and Hill (1989) performed an extensive experimental study of laminar convection for a wide range of modified Ra* ($503 \leq Ra^* \leq 1.75 x 10^7$) in an UHF heated vertical channel. Correlations for local Nu were derived as a function of local Ra* along the channel and temperature variations along the channel's heated wall were measured. Onur and Aktaş (1998) also carried out experiments on natural convection in air between two isothermal plates to study the effect of inclination and plate spacing on the behaviour of heat transfer. They observed that the Nu, which describes the heat transfer to fluid, in all cases was dependent on the separation distance between the plates, but not strongly dependent on the plate inclination. Habib et al (2002) investigated experimentally the natural convection turbulent (Ra > 10^6) airflow between two small vertical plates. The study included both the cases of symmetrically and asymmetrically isothermal heated channels. In the case of symmetrical heating, they observed a high velocity gradient close to the heated plates and a reversed flow at the centre close to the channel entrance. In the second case (asymmetrical heating), the results indicated a large vortex, with an upward flow along the hot plate and downward flow along the cold plate with a wide boundary layer near the hot wall. Habeb (2016) carried out an experimental investigation on 10cm by 10cm vertical channel opened from both sides and derived dimensionless correlations related to heat transfer and airflow rate.

Other applications where a similar heat transfer mechanism found is in passive solar heating collectors. Trombe wall (referred to the French inventor Felix Trombe in the late 1950's) is the best design related to a large scale vertical channel. Many researchers focused their works on this application such as La Pica et al (1993) and Burek, Habeb (2007) and Habeb and Burek (2015) who conducted experimental works on large vertical channel (more than 1m high) and derived some important correlations in regard to heat transfer and airflow characteristics.

However, no studies have been either "encountered or out of the author knowledge" found in the literature to focus on heat transfer by natural convection in channels heated partially from one side. Therefore, the present work was performed on a 1m high vertical duct with variable heat inputs, and heated partially from one side by 10cm by 10cm heating plate. The aim was to derive correlations and characterize heat transfer and airflow under different operating conditions in this type of systems.

EXPERIMENTAL SET-UP:

The test rig consisted of a vertical channel made of steel, open at the top and bottom, and enclosed on the sides, as illustrated in Figure 1.

Figure 1: Test Rig

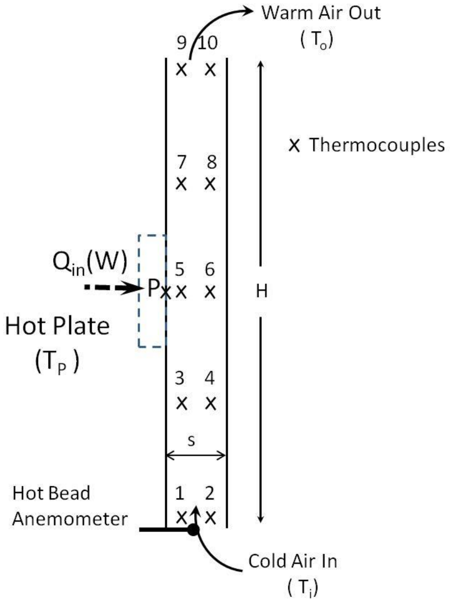

Figure 2: Channel Components

The channel had dimensions of 100cm in height, 10cm in width, and 7cm in depth. To provide the appropriate heat flux, a 10cm by 10cm electric-heating plate made of Aluminium was positioned at the mid-height of the channel. The channel, along with its main components and accessories, was manufactured by Armfield British Company to fulfil the objectives of the current project.

Ten fine thermocouples (0.2mm diameter) of type T copper-constantan were inserted at ten positions along the height of the channel, as shown in Figure 2, to measure the airflow temperature. The temperature of the hot plate was directly read from the power supply monitor. The thermocouples placed in the air stream were centrally located inside lightweight radiation shields made from plastic drinking straws. These shields, approximately 4cm long, were specifically designed to minimize any impact on the airflow or temperatures within the channel. Another thermocouple was used to measure the ambient temperature. All thermocouple wires were connected to a data logger manufactured by Thermo Electric (TE), and their readings were recorded for archival purposes. The thermocouple wires were calibrated at the freezing point (using an ice bath) and boiling point (using a boiling water bath). Assuming a linear relationship between temperature and voltage, each thermocouple was individually adjusted based on its calibration when recording the readings.

Air velocity was measured using a sensitive hot-bead anemometer model TA-5 manufactured by Airflow company. The anemometer was placed at the bottom of the channel to ensure consistent airflow velocity. It provided an analog voltage output that could be recorded by the data logger. The accuracy of the devicexxxx was recently calibrated at BSRIA Instrument Solutions, where it was determined to be ±(2% +0.01m/s)

EXPERIMENTAL PROCEDURE:

The heat input could be adjusted using an adjustable power supply to achieve the desired level of heat input. Five different heat inputs were examined, ranging from 5W to 20W in 5W increments, resulting in a total of four distinct test runs. Each test run began from a cold state, corresponding to the ambient temperature. The electric heating plate was activated, and the heat input was adjusted to the desired level, as measured by a wattmeter. Temperature measurements from all 10 thermocouples were recorded at 5-minute intervals until the system reached a steady state. Once the steady state was achieved (typically around 3 hours after the start of each test run), the airflow's velocity reading was taken to calculate the mass flow rate. Additionally, the ambient air temperature was measured both before and after each test run, at a location near the test rig.

PRELIMINARY RESULTS:

The transient-to-steady-state condition for the temperature data closely matched an exponential model represented by the following mathematical form:

$$y = a\,(1 - \exp^{-bt}) \qquad (1)$$

where:

y – the temperature rise above ambient (˚C).

a – the asymptotic (steady state) temperature (˚C).

b – time constant (1/s).

t – time (s).

The asymptote y from this model was taken as the steady-state value for the purposes of further calculations. SigmaPlot pack version 12.5 was used to achieve the statistic data for the temperature readings and the values of a and b constants. Figure3 shows typical temperature profiles as the system reaches steady state where the all readings were collected for further calculation after about 170 minutes (about 3 hours) operation.

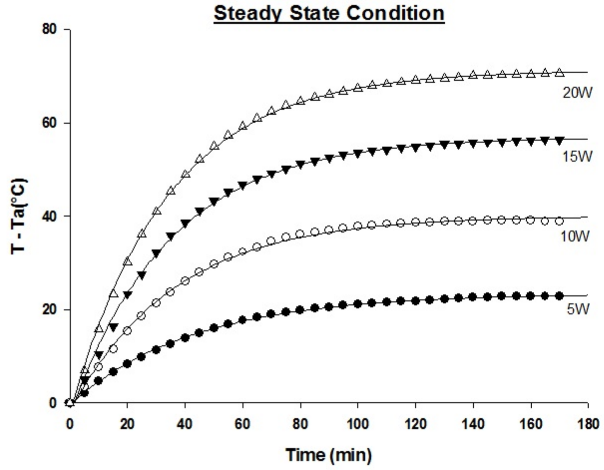

Figure 3: Typical Temperature Rise towards Steady-state Development

TEMPERATURE DISTRIBUTIONS:

Figure 4 depicts the airflow temperature distribution along the channel, using the 5W case as an example. The temperature readings above the ambient temperature (T-Ta) for thermocouples 1 to 10 are plotted against the channel's height. It is evident that temperatures increase along the channel, particularly in the vicinity of the plate, but decrease near the channel exit. According to basic theory, temperatures should continue to rise along the channel's height. However, there is a slight decrease in temperature from approximately 75% of the height and upwards. This phenomenon has been previously observed by Sandberg and Moshfegh (1996), Yilmaz and Gilchrist (2007), and Habeb and Burek (2015), but only along the heated plate. They propose that this drop-off is due to radiation losses directly from the plate and the opposite wall to the ambient through the top of a larger-scale channel. Similar observations were also made regarding the average temperature rise along the channel (as shown in the figure 5), where temperatures increase towards the channel exit and decrease near the exit, attributed to the same aforementioned cause. Figure 6 illustrates the average airflow inside the channel and the average temperature of the heated plate for all operating cases. It is evident that as the heat input increases, both the airflow and heated plate temperatures increase linearly, which is to be expected.

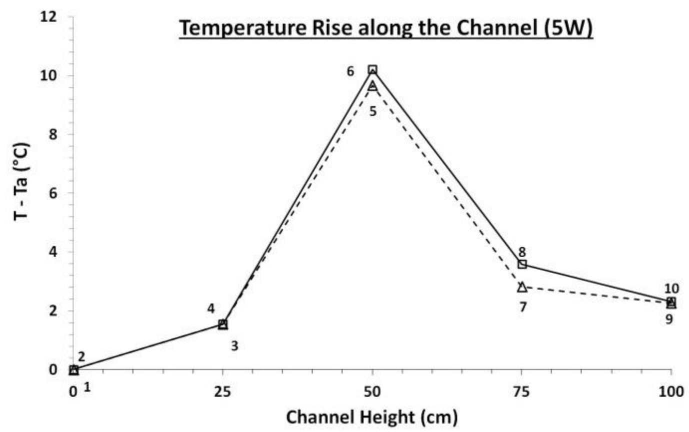

Figure 4: Airflow Temperature Profile along the Channel

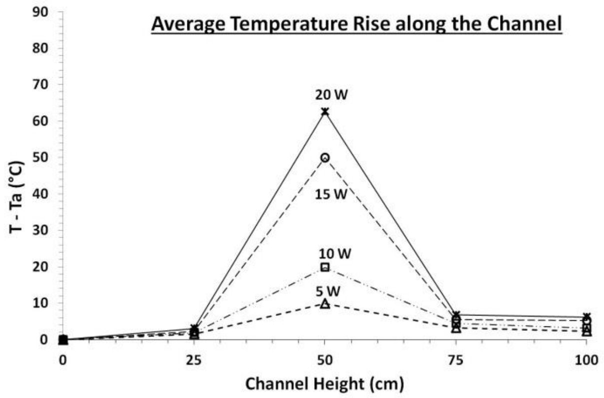

Figure 5: Local Average Airflow Temperature Profile along the Channel

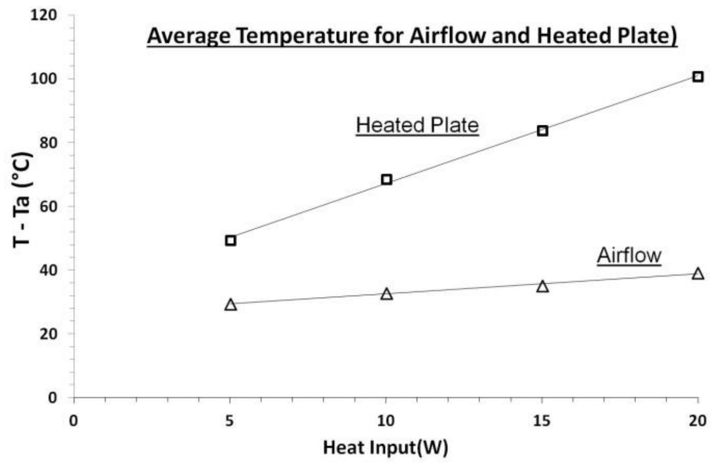

Figure 6: Mean Airflow Temperature Profile along the Channel

VELOCITY AND MASS FLOW RATE:

Throughout each test, air velocity was measured and recorded in the middle of the channel entrance to ensure that the velocity is flat across the channel and thus is equal at any point across the entrance. Figure 7 and Figure 8 show the mean velocity and mass flow rate ,respectively, plotted against the heat input for all heat inputs. Mean velocity and mass flow rate increase as the heat input increases. These results may be expected, though other researchers have observed that for much greater channel depths (between 300mm and 500mm) where the flow rate decreases as channel depth increases (Bouchair et al, 1988) where a reverse flow would occurred. However, no such a phenomena has been observed in the current study.

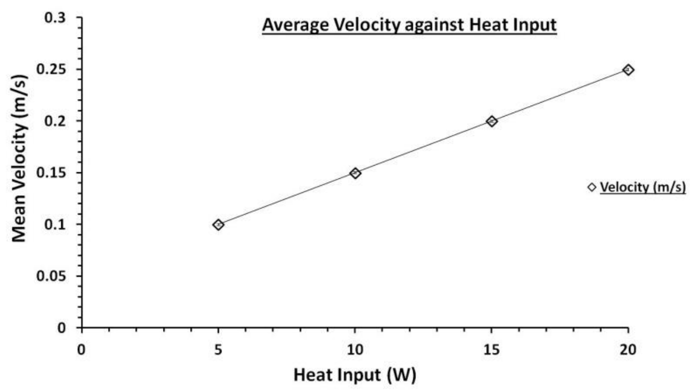

Figure 7: Average Airflow Velocity at Entrance

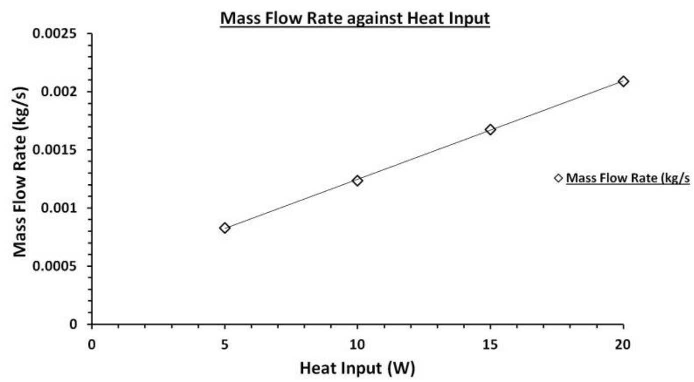

Figure 8: Mass Flow Rate

HEAT TRANSFER COEFFICIENT:

An effective heat transfer coefficient can be calculated for the system by considering the average temperature of the heating plate (T_P) and the average temperature (T_m) of the air in the channel.

$$\bar{h}=\frac{Q_c}{A_p\,(T_p\text{-}T_m)} \qquad (2)$$

Where:

$$Q_c = \dot{m}\, Cp_f\, (T_o - T_i) \qquad (3)$$

Where: $\dot{m} = \rho_i\, A_c\, \bar{u}_i$

The calculated values for $\bar{h}$ are provided in Table 1. It is important to note that these values do not specifically represent the convective heat transfer coefficient between the heated plate and the air. Instead, they take into account all direct and indirect heat transfer mechanisms between the plate and the air, which include radiation from the plate to the channel frame and the resulting convection from the channel frame to the air.

Heat Input (W)	Calculated Heat Transfer Coefficient (W/m^{2}K)
5	9.55
10	12.9
15	17.32
20	21.05

Table 1: Calculated Heat Transfer Coefficient

DATA DIMENSIONLESS CORRELATIONS:

In the present set of experiments, the variation was solely in the heat input. Therefore, it is more appropriate to utilize dimensionless groups that are based on the heat input. One such dimensionless group is the Nusselt number (Nu), which is a dimensionless quantity representing the heat transfer coefficient. The Nusselt number is determined based on the thermophysical properties of the air inside the channel and the channel depth (s).Thus, Nusselt number: $$\mathrm{Nu}(s) = \frac{\bar{h}\, s}{k_m}$$

(4)

- Grashof Number: It is typically defined as the ratio of the buoyancy force to the viscous force in the fluid. It specifically pertains to buoyancy-driven convection. The Grashof number can be encountered in two forms, namely, for uniform wall temperature (UWT).

$$\mathrm{Gr}(H) = \frac{g\, \beta\, \Delta T\; H^3}{\nu^2{}_m} \tag{5}$$

For uniform heat flux (UHF), a commonly employed approach involves a modified form $$\mathrm{Gr}^* = \mathrm{Gr}(H)\mathrm{Nu}(H) = \frac{g\, \beta\, q_c\; H^4}{k_m \nu^2{}_m}$$

(6)

This form was originally introduced by Sparrow and Gregg (1956) and has since been extensively used for UHF applications, including in the present study. This approach leads to the identification of the modified Rayleigh number (Ra*), which is determined by multiplying the modified Grashof number (Gr*) by the Prandtl number (Pr), resulting in the following expression:

Modified Rayleigh number $$\mathrm{Ra}^* = \frac{g\, \beta\, q_c\; H^4}{k_m \nu^2{}_m}\, \mathrm{Pr} \tag{7}$$

-Reynolds number: It is typically utilized in forced flow applications, but in this case, it is also employed to describe the velocity or flow rate of airflow inside the channel since there is no other dimensionless number serving this purpose. This technique has been employed by other researchers, such as LaPica et al. (1993) and Burek and Habeb (2015), to establish correlations with other dimensionless groups. The Reynolds number takes the following form:

$$Re(s) = \frac{\bar{u}_i\, s}{\nu_i} \quad (8)$$

The dimensionless heat transfer coefficient based on the channel depth, denoted as Nu(s), is considered in this study and correlated with Ra*. Mathematical forms are proposed here to represent the dimensionless correlation, which take the following forms:

$$Nu(s) = a\, Ra^{*b} \quad (9)$$

where: a and b are correlations constants.

Figure 7.1 displays LogNu(s) plotted against Ra* for the current study and the work of Habeb (2017) for compression. Regression analysis was conducted, and an empirical equation is presented based on the following relationship:

$$Nu(s) = 0.519(Ra^*)^{0.299} \quad (10)$$

The differences between the two results can be attributed to the size of the channel geometry (s/H), but a similar slope can still be observed.

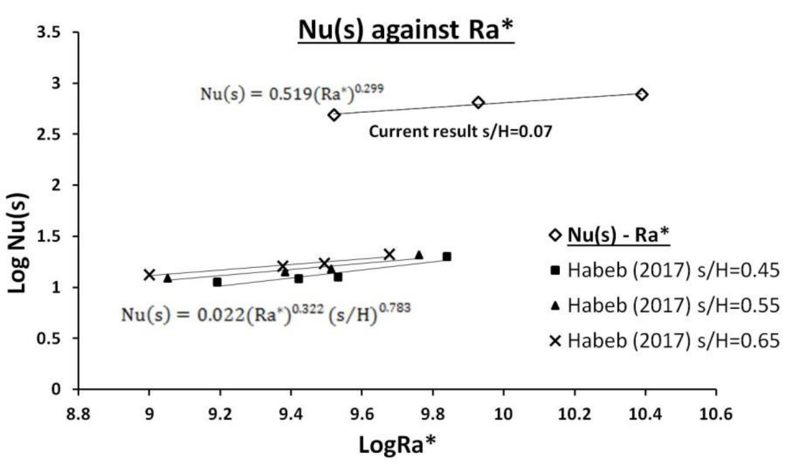

Figure 9: Nusselt Number vs Modified Rayleigh

The constant "b" is remarkably close to the traditional exponent (0.25) described by Azevedo and Sparrow (1985), which is encountered in numerous natural convective heat transfer problems. The statistical data demonstrates a high level of agreement, with a correlation coefficient of 0.99 and a standard error of 0.00812. Similarly, in order to establish a correlation between the velocity or flow rate and Ra*, Re(s) is employed to represent the airflow, as mentioned earlier by LaPica et al. (1993).

Hence, a similar regression technique was employed to derive the following empirical equation, and the results are depicted in the accompanying figure alongside those of Habeb (2017). Although the constant "b" in both results is similar, the differences between the two can be attributed to the same factor mentioned earlier in the Nu correlation. A mathematical form is also proposed, as stated above, to represent the dimensionless correlation, taking the following forms:

$$\mathrm{Re(s)} = \mathrm{a\,Ra^{*b}} \quad (11)$$

where: a and b are correlations constants.

The statistical data is also satisfied which gives 0.99 correlation coefficient and 0.0195 standard error.

$$\mathrm{Re(s)} = 0.033(\mathrm{Ra^*})^{0.33} \quad (12)$$

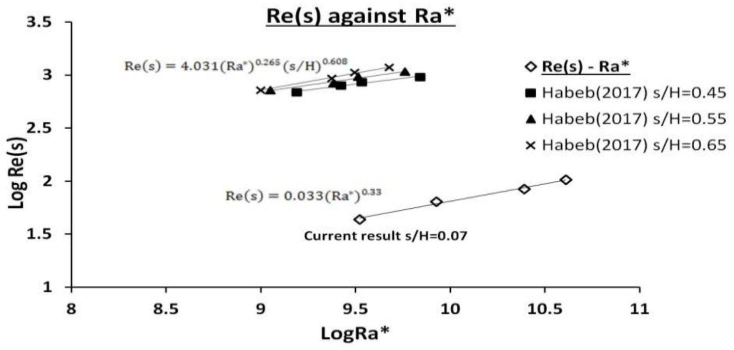

Figure 10: Reynold Number vs Modified Rayleigh

This result is consistent with the theoretical study conducted by Sandberg and Moshfegh (2002), who derived expressions demonstrating that the mass flow rate and velocity are proportional to the cube root of the heat gain in the absence of friction ($\dot{m} \propto Q_c^{1/3}$), which aligns with the findings of the current study. However, their experimental data yielded an exponent value of 0.43 for both open-ended channels, and they attributed this difference to experimental difficulties.

CONCLUSION AND FUTURE WORK:

The data obtained from the present investigation provide valuable insights into the performance of this particular application, wherein the heat input is varied. The following are the main conclusions drawn from this study

- $\bar{h}$ is proportional to $Q_{in}^{0.29}$

and:

- $\dot{m}$ is proportional to $Q_{in}^{0.33}$

Previous findings, such as those by Bouchair (1994), have suggested the existence of an optimal channel depth that may depend on the heat input, resulting in maximum flow rate. However, the data obtained from the current investigation, particularly for low heat inputs and small aspect ratios, do not provide any evidence supporting this claim. In fact, there are indications of possible reverse flow occurring in these conditions.

Further research should be conducted to encompass other crucial components and factors to paint a more comprehensive picture. This could include exploring higher heat inputs, varying channel depths, and utilizing alternative measurement techniques. It is also important to develop and validate a numerical model for this application to reduce time consumption and physical efforts required in the experimental setup.

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